

# Gauge Coupling Unification in GUT and String Models \*

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## Abstract

The results for the running of the gauge couplings in the MSSM are up-dated by proper inclusion of all low scale effects. They are presented as predictions for the strong coupling constant in the scenario with only two parameters at the GUT scale ( $\alpha_U$  and  $M_U$ ) and as a mismatch of the couplings at the scales  $\sim 3 \times 10^{16}$  GeV and  $4 \times 10^{17}$  GeV, when all three couplings are taken as the experimental input.

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1. The gauge coupling unification [1] within the Minimal Supersymmetric Standard Model (MSSM) [2] has been widely publicized as a successful prediction of SUSY–GUTs [2–6]. It is also often discussed in the context of stringy unification, with  $M_{ST} \simeq 4 \times 10^{17}$  GeV [7]. In this paper we up-date the results for the running of the gauge couplings in the MSSM by proper inclusion of all low energy effects such as the best precision of the input parameters at the electroweak scale and the non–logarithmic contribution from the superpartner thresholds [8, 9, 10, 11].

The unification idea is predictive with respect to the behaviour of the  $SU(3) \times SU(2) \times U(1)$  gauge couplings if physics at the GUT scale can be described in terms of only two parameters:  $\alpha_U$  and  $M_U$  (minimal unification). Then we can predict e.g.  $\alpha_s(M_Z)$  in terms of  $\alpha_{EM}(M_Z)$  and  $\sin^2 \theta_W(M_Z)$  (it is worth remembering that  $\sin^2 \theta_W(M_Z)$  and  $\alpha_s(M_Z)$  are at present known with 0.1% and 10% accuracy, respectively). More precisely, the prediction for the strong coupling constant in addition depends on the superpartner spectrum which will, hopefully, be known from experiment. For now, these are free parameters and, denoting them globally by  $T_{SUSY}$  (see the discussion in section 2) we get

$$\alpha_s(M_Z) = F(\sin^2 \theta_W(M_Z), \alpha_{EM}(M_Z), T_{SUSY}) \quad (1)$$

This approach may, however, be too restrictive as it is generally expected that there are non-negligible GUT/string threshold corrections to the running of the couplings (such as heavy threshold and higher dimension operator effects). Then, strictly speaking, all predictivity is lost. However, it is still very interesting to reverse the problem: take the values of all the three couplings at  $M_Z$  as input and use the bottom-up approach to study the convergence of the couplings in the framework of the MSSM. With the same precision calculation and as a function of the SUSY spectrum one can, then, discuss the mismatch of the couplings at any scale of interest and for any value of  $\alpha_s(M_Z)$ , within its 10% experimental uncertainty. It is convenient to introduce the “mismatch” parameters at scale  $Q$ :

$$D_i(Q) = \frac{\alpha_i(Q) - \alpha_2(Q)}{\alpha_2(Q)} \quad (2)$$

and

$$\Delta_i(Q) = \frac{1}{\alpha_i(Q)} - \frac{1}{\alpha_2(Q)} \quad (3)$$

(the latter are directly related to large scale threshold corrections). Of particular interest are  $D_3(M_U)$ , where  $M_U$  is defined as the scale of unification of the  $SU(2) \times U(1)$  couplings (i.e. the scale at which  $D_1 = \Delta_1 = 0$ ), and  $D_i(M_{ST})$ ,  $i = 1, 3$ , with  $M_{ST} = 4 \times 10^{17}$  GeV [12, 7] (and corresponding

$\Delta_i$ s). Clearly, we get this way constraints on physics at the high scale, if it is supposed to have unification and the MSSM as the low energy effective theory. We can also read this information as a hint whether the latter two assumptions look plausible.

In this paper we present our results both as the prediction for  $\alpha_s(M_Z)$  in the minimal unification scenario and as a prediction for the mismatch parameters at  $M_U$  and  $M_{ST}$ , as a function of  $\alpha_s$ .

**2.** We begin with the discussion of the experimental information. Let us first suppose that the (non-supersymmetric) SM is the correct effective theory at the electroweak scale. In this theory the couplings  $g_3, g_2, g_1$  of the  $SU(3) \times SU(2) \times SU(1)$  gauge groups, at  $M_Z$  and in the  $\overline{MS}$  scheme are usually quoted as the values of  $\alpha_{EM}(M_Z)$ ,  $\sin^2 \theta_W(M_Z)$  and  $\alpha_s(M_Z)$ . The electromagnetic coupling constant and the Weinberg angle in the SM are now known with very high precision. The value of  $\alpha_{EM}(M_Z)$  is obtained from the on-shell  $\alpha_{EM}^{OS} = 1/137.0359895(61)$  via the 1-loop RG improved relation [13]:

$$\alpha_{EM}(M_Z) = \frac{\alpha_{EM}^{OS}}{1 - \Delta\hat{\alpha}} \quad (4)$$

where

$$\Delta\hat{\alpha} = 0.0682 \pm 0.0007 + \frac{7\alpha}{2\pi} \log \frac{M_W}{M_Z} - \frac{8\alpha}{9\pi} \log \frac{m_t}{M_Z} \quad (5)$$

The main uncertainty comes from the continuous hadronic contribution to the photon propagator. We explicitly show the top quark mass dependence of  $\alpha_{EM}(M_Z)$ .

The most precise value of  $\sin^2 \theta_W(M_Z)$  in the  $\overline{MS}$  scheme is at present obtained in terms of  $G_F$ ,  $M_Z$  and  $\alpha_{EM}$ . The result depends on  $m_t$  and  $M_{\phi^0}$  (the top and the SM Higgs boson masses respectively) and to a very high precision is given by the following effective formula <sup>1</sup> [9]:

$$\begin{aligned} \sin^2 \theta_W(M_Z) = 0.23166 \pm 0.0003 &+ 5.4 \times 10^{-6} h - 2.4 \times 10^{-8} h^2 \\ &- 3.03 \times 10^{-5} t - 8.4 \times 10^{-8} t^2 \end{aligned} \quad (6)$$

where  $h \equiv M_{\phi^0} - 100$  and  $t \equiv m_t - 165$  (both masses in GeV). The main source of the error is again the hadronic uncertainty in the photon propagator. E.g. for  $m_t = 180$  GeV and  $M_{\phi^0} = 100$  GeV we get  $\sin^2 \theta_W(M_Z) = 0.2312$ .

The value of  $\alpha_s(M_Z)$  is known with much worse precision and depending on the method of determination, the values in the range 0.11-0.13 are quoted

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<sup>1</sup> Notice the small change as compared to ref. [9] which is due to the inclusion of QCD corrections according to ref. [14]. Another, frequently used, fit is given in ref. [4] and its update in [5].

[15]. It is interesting that the lower part of this range is favoured by low energy determinations of  $\alpha_s$  [16, 17] and by a fit to all electroweak data in the framework of the MSSM [19].

Once  $g_i$ s at  $M_Z$  in the SM are extracted from the data, the 2-loop RGE can be used to get them at higher scales. Passing through the thresholds of superpartners the running of the couplings is subject to subsequent modifications of the  $\beta$ -functions [6] with, finally, MSSM RG equations above all the thresholds. Treating the threshold corrections at the 1-loop level (consistently with the 2-loop RGE) this procedure gives <sup>2</sup>:

$$\begin{aligned} \frac{1}{\alpha_i^{MSSM}(Q)} &= \frac{1}{\alpha_i^{SM}(M_Z)} - \frac{C_i}{12\pi} + 2 \sum_k \Delta b_{ik} \log \frac{M_k}{M_Z} \\ &+ 2b_i^{MSSM} \log \frac{Q}{M_Z} + \text{two-loop contribution} \end{aligned} \quad (7)$$

where we have made explicit the  $\overline{MS} \rightarrow \overline{DR}$  conversion factor with  $C_1 = 0$ ,  $C_2 = 2$ ,  $C_3 = 3$  [20].  $M_k$  are the superpartner masses and  $\Delta b_{ik}$  are their contributions to the one-loop  $\beta$  functions of the couplings  $\alpha_i$ . This is the correct result for the running of the gauge couplings at the two-loop accuracy as long as the contribution to the SM from the (non-renormalizable) higher dimension operators, left over after decoupling of superpartners, can be neglected in the process of extracting  $g_i(M_Z)$  from the data (we shall call it the Leading Logarithmic Threshold (LLT) approximation). This requires  $M_k \gg M_Z$  for all superpartner masses.

Assuming that there are only two GUT scale parameters:  $\alpha_U$  and  $M_U$  (i.e. assuming that the potential GUT scale corrections to the gauge coupling unification are negligible) we can predict one of the couplings at  $M_Z$  scale, e.g.  $\alpha_s(M_Z)$  in terms of the other two and of the superpartner masses, which are at present free parameters. In the LLT approximation the dependence of the prediction for  $\alpha_s(M_Z)$  on the supersymmetric spectrum can be described by a single effective parameter  $T_{SUSY}$  [4]:

$$\begin{aligned} \alpha_s(M_Z) &= f(\alpha_1(M_Z), \alpha_2(M_Z), T_{SUSY}) \\ &= \tilde{f}(G_F, M_Z, \alpha_{EM}(M_Z), m_t, M_{\phi^0}, T_{SUSY}) \end{aligned} \quad (8)$$

where [18]:

$$T_{SUSY} = |\mu| \left( \frac{m_{\tilde{W}}^2}{m_{\tilde{g}}^2} \right)^{\frac{14}{19}} \left( \frac{M_{A^0}^2}{\mu^2} \right)^{\frac{3}{38}} \left( \frac{m_{\tilde{W}}^2}{\mu^2} \right)^{\frac{2}{19}} \prod_{i=1}^3 \left( \frac{M_{\tilde{L}_i}^3 M_{\tilde{Q}_i}^7}{M_{\tilde{E}_i}^2 M_{\tilde{U}_i}^5 M_{\tilde{D}_i}^3} \right)^{\frac{1}{19}} \quad (9)$$

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<sup>2</sup> $\alpha_3^{SM}(M_Z)$  differs from  $\alpha_s(M_Z)$  by a threshold correction from the top quark:  $\alpha_3^{-1} = \alpha_s^{-1} + \frac{1}{3\pi} \log(m_t/M_Z)$ .

The effective parametrization in terms of  $T_{SUSY}$  is exact for one-loop RGE and the correction due to the superpartner spectrum then reads:

$$\frac{1}{\alpha_3^{SM}} = \frac{1}{\alpha_3^0} + \frac{1}{2\pi} \frac{19}{14} \log \frac{T_{SUSY}}{M_Z} \quad (10)$$

( $\alpha_3^0$  is the value predicted without the inclusion of threshold corrections). With two-loop equations there is some (weak) dependence on the details of the spectrum through the dependence on the spectrum of the two-loop contribution on the way up to  $M_U$ .

The prediction of the eqns. (7–10) may be subject to important corrections if some of the superpartner masses are  $\mathcal{O}(M_Z)$ . Then the renormalizable SM is not the correct effective theory at the electroweak scale and the non-renormalizable terms should be included when extracting the couplings from the data. Equivalently, we can work at  $M_Z$  in the framework of the full MSSM, extract from the data the MSSM couplings including full 1-loop threshold contribution from SUSY loops (not just the leading logarithms) and study the unification of the MSSM couplings. (Note that in the LLT approximation an equivalent interpretation of equation (7) is:

$$\frac{1}{\alpha_i^{MSSM}(Q)} = \frac{1}{\alpha_i^{MSSM}(M_Z)} + 2b_i^{MSSM} \log \frac{Q}{M_Z} + \text{two-loop contribution} \quad (11)$$

with

$$\frac{1}{\alpha_i^{MSSM}(M_Z)} = \frac{1}{\alpha_i^{SM}(M_Z)} - \frac{C_i}{12\pi} + 2 \sum_k \Delta b_{ik} \log \frac{M_k}{M_Z} \quad (12)$$

where RG running with the MSSM  $\beta$ -functions starts directly from  $M_Z$ , and the threshold corrections are absorbed in a redefinition of  $g_i(M_Z)^{SM} \rightarrow g_i(M_Z)^{MSSM}$ .)

The outlined program has been accomplished by several groups: [9, 10, 11]. Clearly, the values of the MSSM couplings extracted from the data depend now on the superpartner masses <sup>3</sup>  $M_k$ , e.g.:

$$\sin^2 \theta_W(M_Z)^{MSSM} = f(G_F, M_Z, \alpha_{EM}, m_t, M_k, M_{A^0}, \tan \beta)$$

not only by logarithmic terms as in eq.(12), but also by terms  $\mathcal{O}(M_Z/M_k)$  and the additional corrections may be  $\sim 1\%$  for  $\sin^2 \theta_W(M_Z)^{MSSM}$  as shown in ref. [9].

In our analysis we also use  $\alpha_3^{MSSM}$  with the oblique non-logarithmic corrections included [8, 9] but they are unimportant for generic spectra which

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<sup>3</sup>Also  $M_{\phi^0}$  must be replaced by  $M_{A^0}$  and  $\tan \beta$ .

have coloured sparticles rather heavy. We also use the properly extracted  $\alpha_{EM}^{MSSM}$  [9].

The impact of the non-leading SUSY corrections on the prediction for  $\alpha_s(M_Z)$  is illustrated in Fig. 1 for a generic sparticle spectrum obtained in the minimal supergravity model (with universal boundary conditions for the soft SUSY breaking scalar mass parameters at the GUT scale) with radiative electroweak breaking and squark masses below 2 TeV [21]. The results for  $\alpha_s(M_Z)$  are plotted as a function of  $T_{SUSY}$  defined in eq. (9). We compare the results obtained in the LLT approximation for the superpartner thresholds, eq. (7), with their complete inclusion at the one-loop level, as in ref. [9]. The non-leading corrections increase the predicted value of  $\alpha_s(M_Z)$  for  $T_{SUSY} < 100$  GeV. We conclude that unification without GUT threshold corrections predicts e.g. for  $m_t = 160$  (180) GeV  $\alpha_s(M_Z) > 0.126$  (0.128) for  $T_{SUSY} < M_Z$  and  $\alpha_s(M_Z) > 0.121$  (0.123) for  $T_{SUSY} < 300$  GeV.<sup>4</sup> It is clear from eq.(9) that  $T_{SUSY}$  depends strongly on  $\mu$ ,  $m_{\tilde{W}}$ ,  $m_{\tilde{g}}$  and weakly on the other SUSY masses. In models with the GUT relation

$$M_3(M_U) = M_2(M_U) , \quad (13)$$

to a very good approximation:

$$T_{SUSY} \sim \mu \left( \frac{\alpha_2(M_Z)}{\alpha_3(M_Z)} \right)^{\frac{3}{2}} \sim \frac{1}{7} \mu \quad (14)$$

and large  $T_{SUSY}$  means very large higgsino mass. From the naturalness of the Higgs potential [22, 6] it follows then that also the other sparticle masses are to be heavy. For instance in the generic spectrum obtained in models with radiative breaking and universal boundary conditions for the soft scalar masses at the GUT scale  $T_{SUSY} = 300$  GeV corresponds to the squark masses  $\mathcal{O}(2 \text{ TeV})$ . Of course, large values of  $T_{SUSY}$  can be obtained also for small  $\mu$  with a spectrum which violates the GUT relation (13), i.e. with a large ratio  $m_{\tilde{W}}/m_{\tilde{g}}$ . However, it is very difficult to imagine such a scenario without losing the motivation for the minimal unification itself.

**3.** The assumption about negligible GUT scale corrections to coupling unification may be too restrictive. Various groups have discussed the GUT threshold corrections (dependent on the GUT model) [24, 25, 26, 4, 5] and  $\mathcal{O}(M_U/M_{Pl})$  corrections [27, 28]. Admitting non-negligible but strongly model dependent GUT scale corrections means that, strictly speaking, the predictivity is lost and one can only use the bottom-up approach: measure  $g_i(M_Z)$  with better and better precision, measure the sparticle spectrum and study the convergence of the couplings at the large scale. Useful mismatch

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<sup>4</sup> The uncertainty in  $\alpha_s$  induced by the errors in eqs. (6,5) is  $\sim 0.0015$  [9].

parameters then are:  $D_3(M_U)$  and  $\Delta_3(M_U)$  (eqs. (2,3);  $M_U$  is defined by unification of  $\alpha_1$  and  $\alpha_2$ ).  $\Delta_3(M_U)$  is directly related to the GUT threshold corrections. Again, neglecting  $\mathcal{O}(M_Z/M_k)$  non-renormalizable terms, both  $D_3(M_U)$  and  $\Delta_3(M_U)$  are functions of  $\alpha_i$ ,  $i = 1, 2, 3$  and the effective  $T_{SUSY}$ . Inclusion of non-leading supersymmetric threshold corrections brings in additional dependence on the spectrum with, however,  $T_{SUSY}$  still a useful parameter to present the results. We show them as a function of  $T_{SUSY}$  in the LLT approximation for the SUSY thresholds and with their complete inclusion in Fig.2 for our generic spectra for  $m_t = 180$  GeV,  $\tan\beta = 10$  and for three values of  $\alpha_s = 0.11, 0.12, 0.13$ . In Fig.3 we plot the same mismatch parameters as a function of  $\alpha_s$  for our generic spectra. The LLT results for two fixed values of  $T_{SUSY} = 300$  GeV and 1 TeV are also shown for comparison.

The general conclusion is that in the range  $\alpha_s(M_Z) = 0.11 - 0.13$  and  $T_{SUSY} = (20 - 10^3)$  GeV the gauge couplings do unify within the accuracy better than 7% for  $m_t = 160$  GeV and  $\mathcal{O}(8\%)$  for  $m_t = 180$  GeV, with the maximal mismatch for low values of  $\alpha_s(M_Z)$  and  $T_{SUSY}$ . Is this mismatch a lot or a little depends on the GUT model and the expected magnitude of the GUT scale corrections in it [24, 25, 28, 11].

**4.** In stringy unification the unification scale is no longer a free parameter. It is related to the value of the unified coupling [12]:

$$M_{ST} = g_{ST} \times 5.27 \times 10^{17} \text{ GeV} \sim \mathcal{O}(4 \times 10^{17} \text{ GeV}) \quad (15)$$

It is interesting to study within the bottom-up approach the mismatch parameters  $D_3$ ,  $D_1$ ,  $\Delta_3$ ,  $\Delta_1$  (eqs. (2,3)) at the scale  $M_{ST} = 4 \times 10^{17}$  GeV. The results as a function of  $\alpha_s(M_Z)$  are shown in Fig.4. We use again our sample of generic spectra. The results for very heavy spectra with  $T_{SUSY} = 1$  TeV and 5 TeV obtained within the LLT approximation are also shown. The general conclusion which can be drawn from these plots is that the mismatch of the couplings  $\alpha_3$  and  $\alpha_2$  as well as  $\alpha_1$  and  $\alpha_2$  at  $M_{ST}$  is  $> \mathcal{O}(10\%)$ . Therefore, to achieve unification, the string threshold corrections have to be large at the string scale and in addition must conspire so that they are small at the GUT scale, i.e. that the approximate unification occurs at  $M_U \sim 3 \times 10^{16}$  GeV. It is also worth pointing out that the dependence of  $D_1(M_{ST})$  and  $D_3(M_{ST})$  on the supersymmetric spectrum is different and the spectrum which diminishes the first enhances the second.

It is possible to take the attitude that the value of  $\alpha_1$  at the string scale is unconstrained because the Kac-Moody level of the U(1) group can be treated as a free parameter [7]  $k_1$ .<sup>5</sup> We then have:

$$k_1 \alpha_1 = \alpha_2 = \alpha_3 \quad (\text{at } M_{ST}) \quad (16)$$

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<sup>5</sup> In our convention  $k_1 = 1$  in the case of SU(5) - type unification. This differs from the definition adopted in ref. [7].

In this case (and for negligible stringy threshold corrections) our parameter  $D_1$  is related to the parameter  $k_1$ :

$$k_1 = \frac{1}{D_1 + 1}. \quad (17)$$

With our generic spectra we get:  $k_1 = 0.88 - 0.92$ . However, even then we are still faced with a large mismatch between  $\alpha_3$  and  $\alpha_2$  which for our generic spectra requires large string threshold corrections.

Finally it is interesting to go beyond the discussion based on our generic spectra and to address the following two questions:

1) Does there exist a pattern of the MSSM spectrum which shifts the unification point of all three couplings to  $M_{ST}$  with negligible stringy thresholds?

2) Suppose  $k_1 \neq 5/3$  (i.e.  $\alpha_1 \neq \alpha_2$  at  $M_{ST}$ ) and helps to unify  $\alpha_1$  and  $\alpha_2$ . Are there MSSM spectra which unify  $\alpha_3$  and  $\alpha_2$  at  $M_{ST}$  with negligible stringy thresholds?

In order to answer these questions it is useful to introduce two new effective parameters describing the impact of the SUSY spectrum on unification of  $\alpha_1$  and  $\alpha_2$  and  $\alpha_2$  and  $\alpha_3$  separately. From eqs. (11) and (12) we have:

$$M_U = M_U^0 \left( \frac{M_Z}{T'_{SUSY}} \right)^{\frac{2}{7}} \quad (18)$$

where  $M_U^0$  ( $M_U^0$ ) is the crossing point of  $\alpha_1$  and  $\alpha_2$  with SUSY threshold corrections included (neglected) and

$$T'_{SUSY} = \left( M_{A^0} \mu^4 m_{\tilde{W}}^{20} \right)^{\frac{1}{25}} \frac{\left( M_{\tilde{Q}}^7 M_{\tilde{L}} \right)^{\frac{1}{8}}}{\left( M_{\tilde{U}}^4 M_{\tilde{D}} M_{\tilde{E}}^3 \right)^{\frac{1}{8}}} \quad (19)$$

All generations have the same masses in the above formula but a generalization is straightforward. Noticing that all (none) of the sparticles in the denominator (numerator) are SU(2) singlets one can write a simplified formula:

$$T'_{SUSY} = \frac{M_L^2}{M_R} \quad (20)$$

with obvious definitions of the averages  $M_L$  and  $M_R$ . In Fig.5 we plot  $M_U$  resulting from formula (18). For  $M_U = 4 \times 10^{17}$  we need  $T'_{SUSY} = 2 \times 10^{-2}$  GeV which means that for  $M_L = M_Z$  we would have  $M_R = 400$  TeV. As we can see the answer to question 1 is negative. Regardless of the  $\alpha_2 - \alpha_3$  unification, bringing  $M_U$  up to  $M_{ST}$  would require an unacceptable  $M_R$ .



Turning to question 2 we study the correction to  $\alpha_3^{SM}(M_Z)$  predicted from the condition  $\alpha_3(M_{ST}) = \alpha_2(M_{ST})$ , induced by the SUSY thresholds. From (11) and 12) we obtain:

$$\frac{1}{\alpha_3^{SM}} = \frac{1}{\alpha_3^0} + \frac{1}{2\pi} \frac{19}{6} \log \frac{M_D}{M_S} \quad (21)$$

Where

$$\frac{M_D}{M_S} = \left( \frac{M_{A_0}}{M_Z} \right)^{\frac{1}{19}} \left( \frac{m_{\tilde{W}}^8 \mu^4}{m_{\tilde{g}}^{12}} \right)^{\frac{1}{19}} \left( \frac{M_{\tilde{Q}}^3 M_{\tilde{L}}^3}{M_{\tilde{U}}^3 M_{\tilde{D}}^3} \right)^{\frac{1}{19}} \quad (22)$$

and  $\alpha_3^0$  is the value predicted without the inclusion of SUSY threshold corrections. In Fig.6 we show  $\alpha_3$  predicted with the use of formula (21) (with  $\alpha_3^0$  obtained from the two-loop RGE) for  $M_{ST} = (3.5, 4.0, 4.5) \times 10^{17}$  GeV as a function of the ratio  $M_D/M_S$ . In order to get  $\alpha_s < 0.13$  one needs  $M_D/M_S > 20$ . At this point we disagree with the recent analysis of ref. [29] which reconciles  $\alpha_s(M_Z) = 0.118$  with the stringy unification for the SUSY spectra with smaller hierarchies. Taking masses in the numerator of our eq. (22)  $\sim 30$  times larger than masses in the denominator we still get  $\alpha_s \geq 0.125$  as can be seen from Fig.6<sup>6</sup>. This disagreement is mainly due to the use of one-loop RGE in ref. [29] and somewhat higher value of  $\sin^2 \theta_W(M_Z)$  (more appropriate for  $m_t \sim 160$  GeV). The ratio  $M_D/M_S$  is dominated by the ratio  $m_{\tilde{W}}^2 \mu / m_{\tilde{g}}^3$  which in the case of string unification is more model dependent than for GUTs [30, 31]. In particular it is conceivable in this case that  $m_{\tilde{W}} > m_{\tilde{g}}$ . For  $M_S = M_Z$  we would get  $M_D > 2$  TeV but in fact for so light spectrum the non-logarithmic corrections could raise the predicted value of  $\alpha_s$  as is evident from Fig.1. For  $M_S = 150$  GeV, when non-logarithmic effects are small, we get  $M_D > 3$  TeV. We conclude that it is possible to raise the  $\alpha_2 - \alpha_3$  unification scale up to  $M_{ST}$  but only with highly unnatural SUSY spectra, with the heaviest sparticles above 3 TeV and with  $\alpha_s(M_Z) \simeq 0.13$ . Otherwise large string threshold corrections are needed.

**5.** We have discussed the impact of SUSY thresholds on the unification of gauge couplings in the framework of GUT and string theories. Non-logarithmic SUSY corrections can be important for the phenomenologically interesting case of light superpartners. These corrections always reduce the value of  $\sin^2 \theta_W(M_Z)^{MSSM}$  which in turn raises the value of  $\alpha_s(M_Z)^{SM}$  predicted from SUSY unification. In the minimal unification scenario (i.e. with negligible GUT scale corrections to the running of the couplings) one gets  $\alpha_s(M_Z) > 0.121(123)$  for the effective parameter  $T_{SUSY} < 300$  GeV

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<sup>6</sup>For  $\alpha_s(M_Z) \simeq 0.12$  the masses should be split by a factor of at least 60.

and  $\alpha_s(M_Z) > 0.115(117)$  for  $T_{SUSY} < 1$  TeV, for  $m_t = 160$  and 180 GeV, respectively. For the generic spectra in the minimal supergravity model  $T_{SUSY} \sim 1$  TeV corresponds to very heavy sfermions, e.g. squark masses are  $\mathcal{O}(5 \text{ TeV})$ . More generally in the bottom-up running the couplings do unify within a few percent accuracy even for low  $\alpha_s(M_Z)$  and small values of  $T_{SUSY}$ , e.g. the mismatch between  $\alpha_3$  and  $\alpha_2$  at the scale of unification of  $\alpha_1$  and  $\alpha_2$  is generically below  $\mathcal{O}(5\%)$ . The mismatch of the couplings at  $M_{ST} = 4 \times 10^{17}$  GeV is much larger, typically  $\mathcal{O}(10\%)$  or more, and it cannot be eliminated by any sensible superpartner spectrum. String unification requires, therefore, large string threshold corrections (which, however, may not be unrealistic [32]) which conspire to give the effective unification scale  $\sim 3 \times 10^{16}$  GeV. The scenario with  $\alpha_1$  and  $\alpha_2$  unified by treating the Kac-Moody level  $k_1$  as a free parameter is not particularly helpful with regard to the coupling unification at  $M_{ST}$  (and is rather uneconomical).

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## FIGURE CAPTIONS

### Figure 1.

$\alpha_s(M_Z)$  predicted by the minimal unification as a function of  $T_{SUSY}$  for different values of  $m_t$  and  $\tan\beta$ . Squares (stars) correspond to the LLT (full) calculation of the supersymmetric thresholds for a generic sample of SUSY spectra obtained in the minimal supergravity model with radiative electroweak breaking and universal boundary conditions.

### Figure 2.

Mismatch parameters  $\Delta_3(M_U)$  and  $D_3(M_U)$  for  $m_t = 180$  and  $\tan\beta = 10$  as a function of  $T_{SUSY}$  for several values of  $\alpha_s$ . Squares (stars) correspond to the LLT (full) calculation of the supersymmetric thresholds. Sample of spectra as in Fig.1c. Solid lines extrapolate  $\Delta_3(M_U)$  and  $D_3(M_U)$  in the LLT approximation up to  $T_{SUSY} = 1$  TeV.

**Figure 3.** Mismatch parameters  $\Delta_3(M_U)$  and  $D_3(M_U)$  as a function of  $\alpha_s$  for the same sample of spectra as in Fig.1c. Squares, stars and circles show the results of the full calculation for spectra with  $M_{\tilde{Q}} < 500$  GeV,  $500 \text{ GeV} < M_{\tilde{Q}} < 1$  TeV and  $1 \text{ TeV} < M_{\tilde{Q}} < 2$  TeV, respectively. For comparison the LLT calculation for a spectra with  $T_{SUSY} = 300$  GeV (1 TeV) are marked by the solid (dashed) lines.

### Figure 4.

Mismatch parameters  $\Delta_3$ ,  $D_3$ ,  $\Delta_1$  and  $D_1$  at the string scale  $M_{ST} = 4 \times 10^{17}$  as a function of  $\alpha_s$  for our generic spectra. Markers as in Fig.3. Solid, dashed and dash-dotted lines correspond to the LLT calculation for  $T_{SUSY} = 300$  GeV, 1 and 5 TeV respectively.

**Figure 5.**  $M_U$  as a function of  $T'_{SUSY}$  from eq. (18) for  $m_t = 180$  GeV.

### Figure 6.

Prediction for  $\alpha_s$  from the condition  $\alpha_3(M_{ST}) = \alpha_2(M_{ST})$  with  $M_{ST} = 3.5$ , (solid) 4.0 (dashed) and  $4.5 \times 10^{17}$  GeV (dash-dotted) with the use of eq. (21) as a function of the parameter  $M_D/M_S$  for  $m_t = 180$  GeV.

Figure 1.

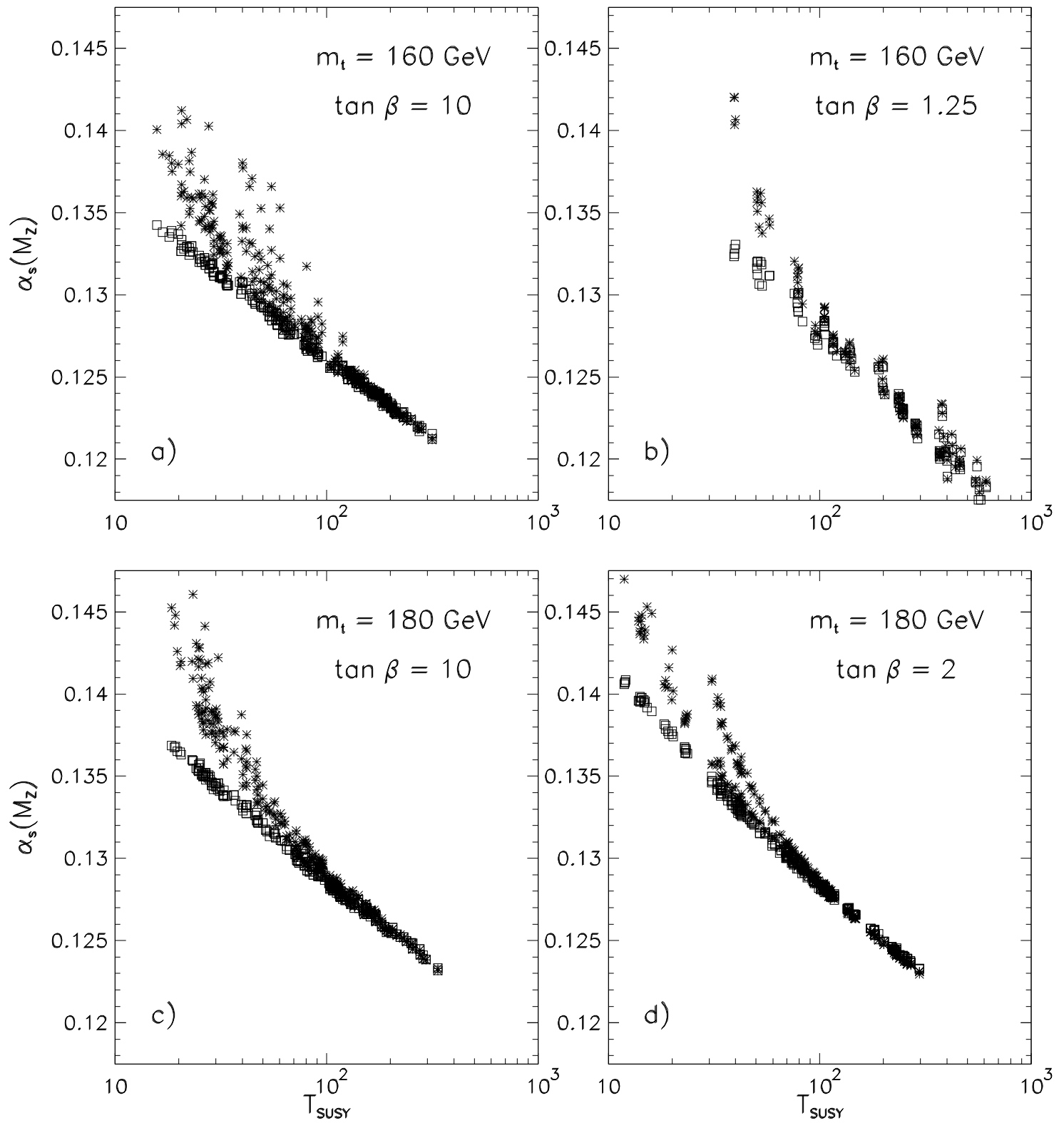


Figure 2

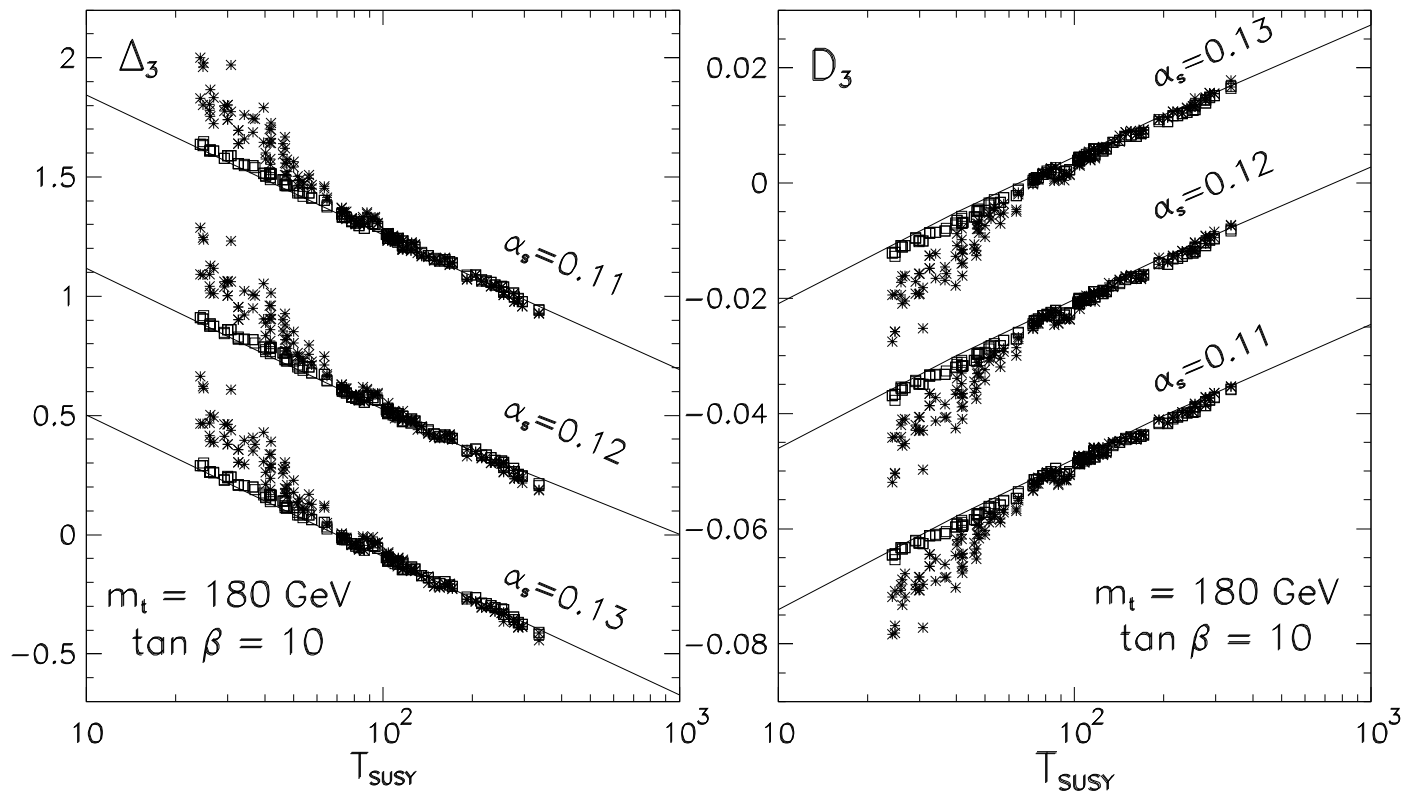


Figure 3

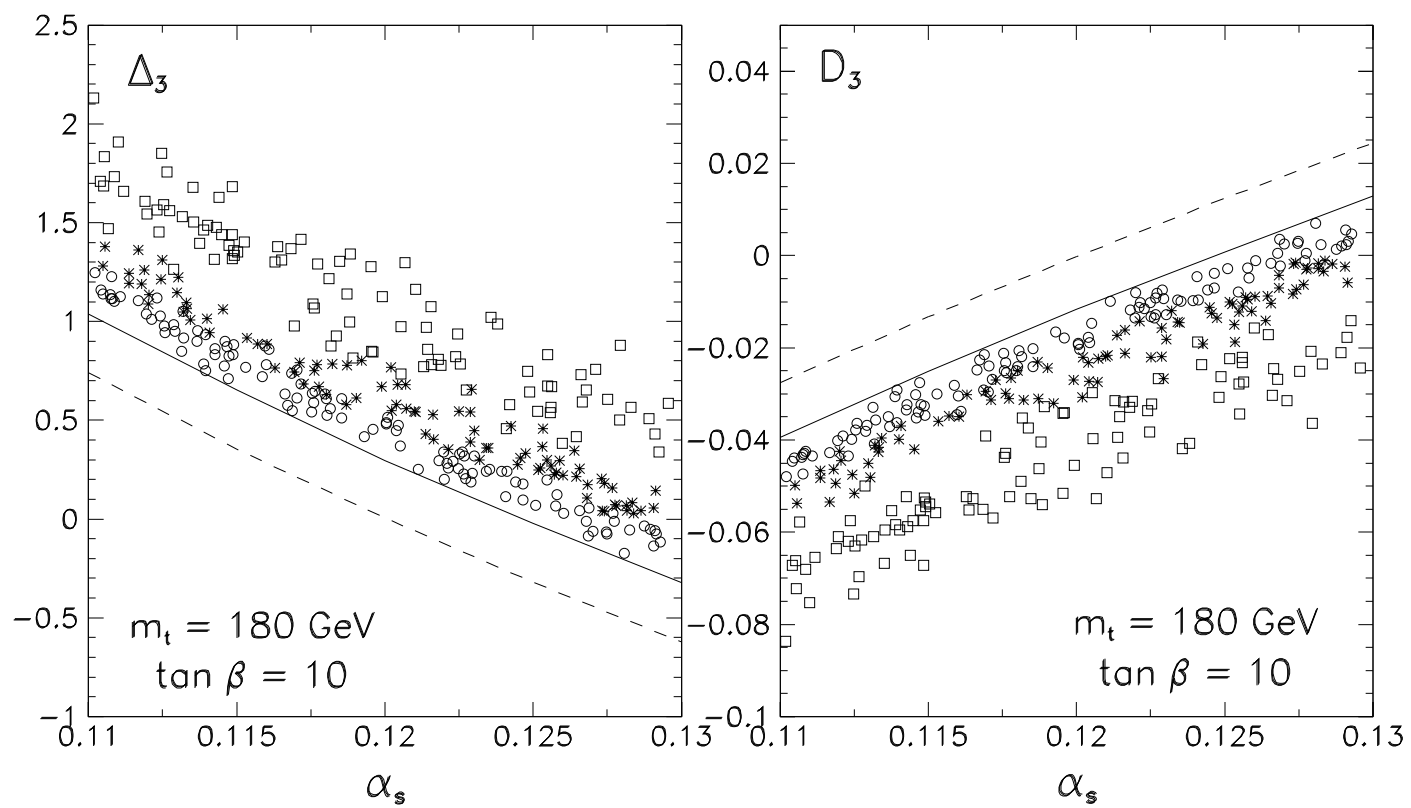




Figure 4

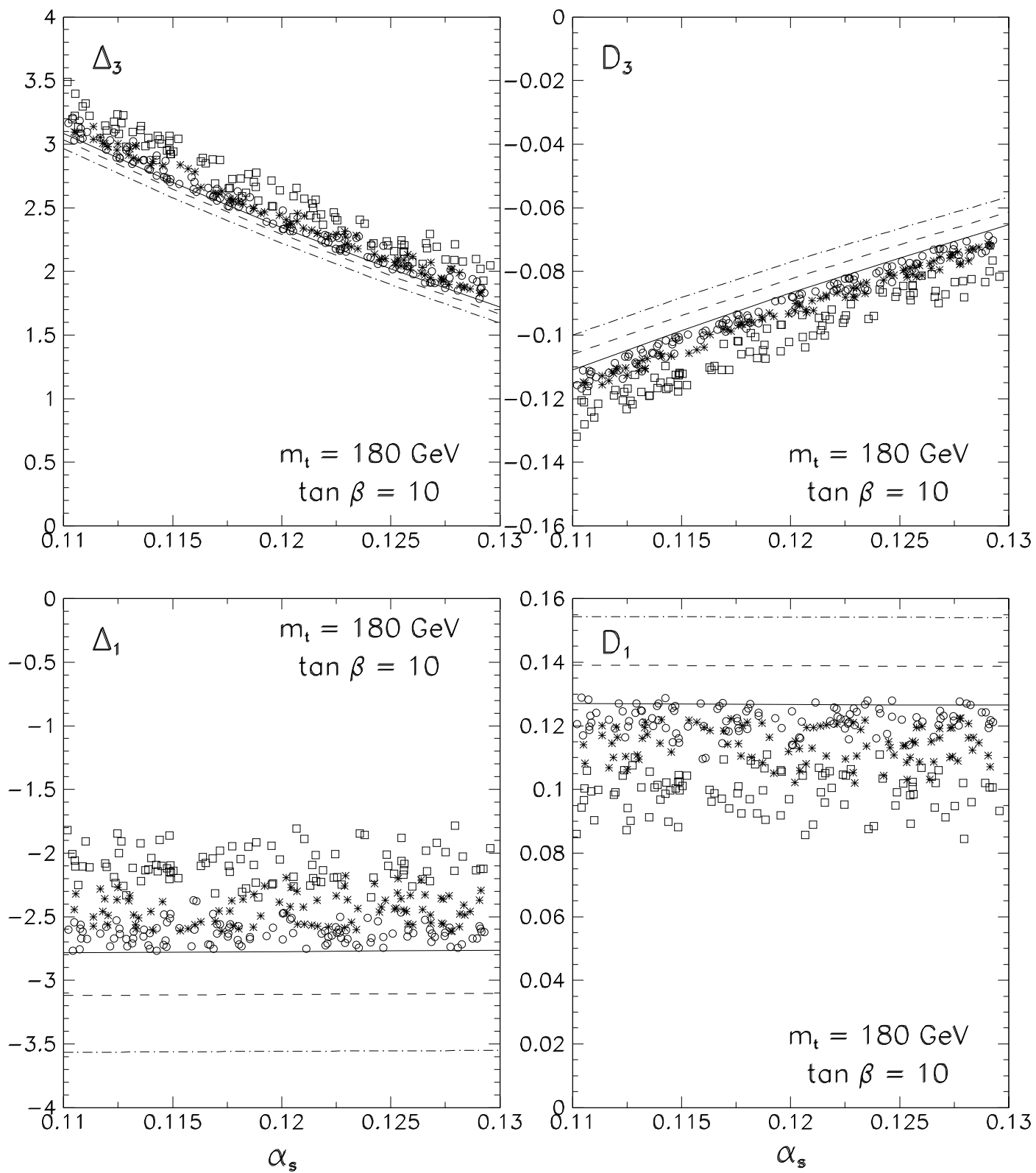


Figure 5

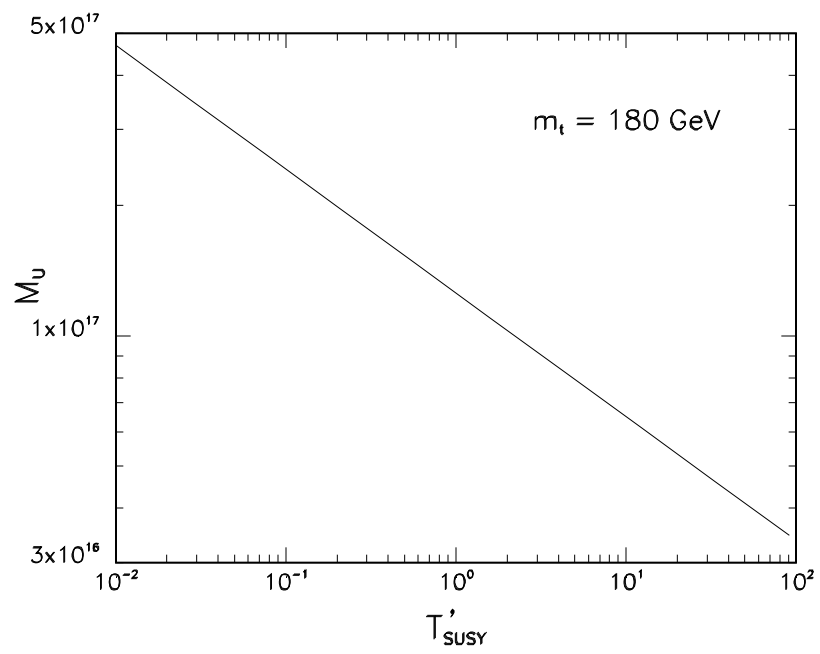


Figure 6

